Measuring B_s Width Difference at the $\Upsilon(5s)$ Using Quantum Entanglement

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About 90% of $B_s\overline{B}_s$ pairs produced at the $\Upsilon(5s)$ resonance are initially $B_s^*\overline{B}_s^*$ pairs which decay radiatively to $B_s\overline{B}_s$. This implies that the $B_s\overline{B}_s$ pair will then be in an eigenstate of charge conjugation (i.e. $C = -1$) and therefore in an entangled state. This allows for a determination of $\Delta\Gamma_s/\Gamma_s$ and the CP phase using a number of possible correlations between the decays of the two B_s mesons. In particular, we consider the time integrated correlation; the time ordering asymmetry and the time ordering-charge asymmetry, which in addition to time ordering distinguishes B_s from $\overline{B}_s$, for various combinations of final states. With the statistics of about $O(10^7 - 10^8)$ $\Upsilon(5s)$ events available at B factories, we find that the time ordering asymmetry between suitably defined hadronic and flavor specific (tagging) decays offers a promising method for determining the width difference. The corresponding time ordering-charge asymmetry can also bound the mixing phase. Similar observables involving exclusive decays are also considered. At the super B factories with O(50) times greater luminosity time ordering and time ordering-charge asymmetries between inclusive and exclusive modes may also provide additional bounds on the phases in those decays.

I. INTRODUCTION

Resonance production of B-mesons at electron-positron colliders have proven to be an extraordinarily effective tool to study flavor oscillations. The B-factories to date have largely only used the $\Upsilon(4s)$ resonance where the only neutral B-meson produced is the B_d. To produce the more massive B_s meson it is necessary to operate at the $\Upsilon(5s)$ peak. Indeed this mode of operation has already been carried out at CLEO[1] and more recently at BELLE [2, 3] where 100fb^{-1} of data, amounting to about 10^7 $B_s\overline{B}_s$ pairs has been collected.

In the last few years, much progress has been made in the study of mixing in the B_s system at D0 [4] and CDF [5]; the mass difference and width differences have been measured and some bounds have been placed on the oscillation phase [6, 7]. Further progress is expected in the future and also at the LHCb experiment [8] when it takes data. While these experiments offer the advantage of high event rates, they are limited in the number of final states which they can observe. If new physics contributes to the mixing or decay in the B_s system, it could result in different CP phases in different final states so doing mixing studies in a larger sample of final states is desirable.

At a B factory the detector environment is generally cleaner than at a hadronic machine so that a larger set of final states can be observed, particularly those involving neutrals in their final state. On the other hand the B factory has the disadvantage that the B_s mesons are less boosted so that oscillations with frequency Δm_s might not be observable. Finally, the B-factory does have the feature that the mesons are produced in correlated pairs. The focus of this paper will be how to exploit this unique feature to obtain information about B_s mixing.

In this paper we will consider how the quantum correlations between the B_s mesons allow the determination of the width difference of the B_s system through the correlation of inclusive final states between the two meson decays. Further studies of the correlations with exclusive or semi-exclusive (*i.e.* an exclusive state with several quantum amplitudes or polarizations) final states could further give information about the width difference and the CP phases.

We will show that luminosity sufficient to produce $10^7 - 10^8$ will generate sufficient statistics to carry out such studies however systematic errors originating from the single B_s branching ratios would need to be addressed in order to obtain precision results.

Another way to address this problem is to take advantage of the correlated time evolution of the entangled meson pair. We find that time ordering asymmetries between different inclusive states are particularly sensitive to $\Delta\Gamma_s$. A related time ordering-charge asymmetry can also give the tangent of the mixing phase, in spite of the fact that we assume the Δm_s oscillations are too rapid to be observed in detail at B factories. This capability will help resolve the sign ambiguities in the time ordering asymmetry and correlation measurements which only give the cosine of phases. This methodology can be extended to correlations between inclusive and semi-exclusive states to obtain separately the CP phase for those decays. The key advantage of using these asymmetries is that they are null experiments which vanish in the limit of no mixing and therefore are not subject to large systematic errors from input branching ratios.

Future B factories are being considered with luminosities 50 times the current machines[9–11]. If a fraction of this

luminosity is on the $\Upsilon(5s)$ then mixing effects in a larger set of exclusive decays may be probed including final states which are more sensitive to new physics.

This method is complementary to the methods used at hadronic experiments, in particular CDF and D0 where the oscillations proportional to Δm_s can be resolved. Note in particular the existing results from D0 and CDF[4, 5, 12].

In this paper we will generally be considering observables which should be within the capabilities of modern B-factories. For instance a B-factory with a luminosities of $O(10^{34}\text{cm}^{-2}s^{-1})$ operating at the $\Upsilon(5s)$ peak produces $b\bar{b}$ final states with a cross section[2] of (0.302 ± 0.014)nb. Thus, running 1 year there will be $O(10^8)$ such events. Upgrade plans for KEK have as the design luminosity 50 ab^{-1}. If about 10% of this luminosity is delivered at the $\Upsilon(5s)$ then this will produce 1.5×10^9 $b\bar{b}$ events.

There are several compelling motivations to preform these measurements. First of all, recent theoretical progress [13–16] suggests that a fairly robust Standard Model prediction of the width difference may be possible and so measurement of this quantity could become a good test of the Standard Model. The Standard Model also predicts that the mixing and decay phases in the B_s system are small, and the same for all final states with quark content $c\bar{c}s\bar{s}$ since the overall phase of this combination is $arg(-(V_{tb}V_{ts}^*)/(V_{cb}V_{cs}^*)) \approx 0.02$. Thus the observation of a significant phase and/or variation in the phase between different $c\bar{c}s\bar{s}$ final states would indicate new physics.

In Section II we discuss the The $B_s\overline{B}_s$ correlated state, oscillation and CP violation in decays are dealt with in Section III. Inclusive and exclusive final states are discussed in Section IV, time independent and time dependent effects follow in Sections V and VI respectively. A brief conclusion is given in Section VII.

II. THE $B_s\overline{B}_s$ CORRELATED STATE

At the $\Upsilon(5s)$ peak there is sufficient energy to produce the final states $B_s\overline{B}_s$, $B_s^*\overline{B}_s$, $B_s\overline{B}_s^*$ and $B_s^*\overline{B}_s^*$. The vector state decays radiatively, $B_s^* \to \gamma B_s$, so in all of these cases the final state consists of a $B_s\overline{B}_s$ pair and $n = 0, 1$ or 2 photons. This means that the charge conjugation of the final $B_s\overline{B}_s$ system is directly related to the number of photons so radiated[17]. In particular, the initial $\Upsilon(5s)$ state is in a $C = -1$ state so if it goes directly to $B_s\overline{B}_s$ then the meson pair must also be in a $C = -1$ state. If the transition is through $B_s^*\overline{B}_s$ or $\overline{B}_s^*B_s$ then the final $B_s\overline{B}_s$ pair must be in a $C = +1$ state because of the associated photon. Likewise if the transition is through $B_s^*B_s^*$ then the final state consists of two photons and a $B_s\overline{B}_s$ pair so the $B_s\overline{B}_s$ pair is in a $C = -1$ state. Note that this same argument applies if the meson pair is produced through a virtual photon (indeed one cannot a priori separate the two channels). Current results[2] indicate that about 90% of $B_s\overline{B}_s$ pair production is through the $B_s^*B_s^*$ (while the relative contributions of the other two modes are less well measured) so the final meson pair is in the C= -1 state at least 90% of the time.

If we let $\vec{k}$ be the 3-momentum of the B_s in the center of mass of the $B_s\overline{B}_s$ pair, then the wave function is constrained by the fact that for a scalar anti-scalar pair P=C=$(-1)^L$ so that for the odd L, C=-1 case:

$$\Psi^{C=-1} \propto \frac{1}{\sqrt{2}} \left(|B_s(\vec{k})\rangle|\overline{B}_s(-\vec{k})\rangle - |\overline{B}_s(\vec{k})\rangle|B_s(-\vec{k})\rangle \right) \tag{1}$$

while for the even L, C=$+1$ case the wave function is:

$$\Psi^{C=+1} \propto \frac{1}{\sqrt{2}} \left(|B_s(\vec{k})\rangle|\overline{B}_s(-\vec{k})\rangle + |\overline{B}_s(\vec{k})\rangle|B_s(-\vec{k})\rangle \right) \tag{2}$$

Let us denote $a = \sigma(e^+e^- \to B_s^{(*)}\overline{B}_s^{(*)})/\sigma(e^+e^- \to b\bar{b})$ at the $\Upsilon(5s)$ peak and let r, r^* and r^{**} refer to the fraction of this branching ratio which contains 0, 1 or 2 vectors respectively. Current experimental results [1, 2, 18] give $a = 19.7 \pm 2.9\%$ and $r^{**} = 90.1^{+3.8}_{-4.0}\%$. r and r^* are as yet unmeasured but are constrained by the r^{**} result since $r + r^* + r^{**} = 1$. Clearly then the $B_s\overline{B}_s$ pair is dominantly in the $\Psi^{C=-1}$ state, at least 90% of the time.

The $B_s\overline{B}_s$ pairs thus produced at the $\Upsilon(5s)$ in the C=-1 state are therefore similar to the $B^0\overline{B}^0$ pairs produced at the $\Upsilon(4s)$. However, because of the different regime of mixing parameters, the quantities which can be measured using this effect are somewhat different.

III. OSCILLATION AND CP VIOLATION IN B_s DECAY

Let us now turn our attention to the time evolution of the B_s mesons. Unlike in the B_d case, direct measurement of the oscillations driven by the mass difference are probably not practical at a B-factory since[18] $\Delta m_s = 17.77 \pm$

$0.1 \pm 0.07 \text{ps}^{-1}$ is so large, it gives rapid oscillations which are hard to resolve. In this paper we will instead focus on results related to the component of mixing driven by the width difference. Currently the width difference is not well measured, results from CDF[6] and D0[7] give [18] $|\Delta\Gamma_s| = 0.154^{+0.054}_{-0.070}\text{ps}^{-1}$. This is about 20% of the B_s decay rate where $\tau_s = 1/\Gamma_s = 1.472^{+0.024}_{-0.026}\text{ps}$. As we shall show, this range of mixing parameters allows time integrated and time dependent studies to provide information concerning the width difference and CP violation in the B_s system.

Let us first review the standard formalism for time evolution in a single neutral meson[19] and then generalize this to an entangled pair of mesons. We denote a state $\Psi = a_\Psi|B_s\rangle + \overline{a}_\Psi|\overline{B}_s\rangle$ which is a mixture of B_s and $\overline{B}_s$ by:

$$\Psi = \begin{bmatrix} a_\Psi \\ \overline{a}_\Psi \end{bmatrix} \tag{3}$$

In this basis we can write the the general mass matrix subject to CPT constraints:

$$M = \begin{bmatrix} A & -p^2 \\ -q^2 & A \end{bmatrix} \tag{4}$$

Here A, q and p are general complex numbers where we take the convention that $Re(pq) > 0$. This matrix then has complex eigenvalues $\mu_1 = A - pq$ and $\mu_2 = A + pq$. We will write these eigenvalues as $\mu_1 = m_1 - \frac{i}{2}\Gamma_1$ and $\mu_2 = m_2 - \frac{i}{2}\Gamma_2$ where $m = \frac{1}{2}(m_1 + m_2)$ and $\Gamma_s = \frac{1}{2}(\Gamma_1 + \Gamma_2)$. The eigenfunctions corresponding to these two eigenvalues are:

$$|\Psi_1\rangle \equiv |B_1\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \begin{bmatrix} p \\ q \end{bmatrix} \qquad |\Psi_2\rangle \equiv |B_2\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \begin{bmatrix} p \\ -q \end{bmatrix} \tag{5}$$

If $\Psi(t)$ is the state at time t, this is related to the state at time=0 by:

$$\Psi(t) = U(t)\Psi(0) \tag{6}$$

where the time evolution matrix U satisfies

$$i\frac{d}{dt}U(t) = MU(t) \tag{7}$$

The solution to this equation is:

$$U = \begin{bmatrix} g_+ & e^{-\frac{1}{2}\eta - i\phi}g_- \\ e^{\frac{1}{2}\eta + i\phi}g_- & g_+ \end{bmatrix} \tag{8}$$

where $e^{\frac{1}{2}\eta + i\phi} = q/p$ and $g_\pm = \frac{1}{2}\left(e^{-i\mu_1 t} \pm e^{-i\mu_2 t}\right)$.

In the B_s system $|q/p| \approx 1$, the current experimental value is[18] $|q/p| = 1.0019 \pm 0.0047$. This deviation from $\eta = 0$ is undetectably small for the methods we will discuss in this paper so we will proceed taking the approximation that $\eta \approx 0$. This experimental value of $|q/p|$ is obtained from measurement of the semi-leptonic asymmetry,

$$A_{SL}(B_{s,d}) = \frac{N(\overline{B}_{s,d} \to \ell^+\nu_\ell X) - N(B_{s,d} \to \ell^-\overline{\nu}_\ell X)}{N(\overline{B}_{s,d} \to \ell^+\nu_\ell X) + N(B_{s,d} \to \ell^-\overline{\nu}_\ell X)} \tag{9}$$

using the relation

$$e^{\frac{1}{2}\eta} = |q/p| = \left(\frac{1 - A_{SL}}{1 + A_{SL}}\right)^{1/4} \tag{10}$$

The average value of $A_{SL}(B_s)$ used in obtaining this value of $|q/p|$ is $A_{SL}(B_s) = -0.0037 \pm 0.0094$ where the authors of [18] have combined $\Upsilon(4s)$ data giving $A_{SL}(B_d)$ with Tevatron data which gives a linear combination of $A_{SL}(B_d)$

with $A_{SL}(B_s)$. A more resent D0 result [20] which is not included in this average gives the combined asymmetry as $A_{SL}^b = -0.00957 \pm 0.00251(stat) \pm 0.00146(syst)$ where

$$A_{SL}^b = \frac{N(b \to \ell^+ \nu_\ell X) - N(\overline{b} \to \ell^- \overline{\nu}_\ell X)}{N(b \to \ell^+ \nu_\ell X) + N(\overline{b} \to \ell^- \overline{\nu}_\ell X)}. \tag{11}$$

This is a linear combination of $A_{SL}(B_d)$ and $A_{SL}(B_s)$ which has a 3.2 sigma discrepancy from the Standard Model prediction for this quantity: $A_{SL}^b = -2.3^{+0.5}_{-0.6} \times 10^{-4}$ (predicted). This deviation highlights another utility of $\Upsilon(5s)$ B-factories which should be able to directly measure A_{SL}^b as well as the B_s and B_d components separately and thus clarify the comparison of theory to experiment in A_{SL}.

Let f_i be a single quantum state with A_i and $\overline{A}_i$ being the decay amplitudes of B_s and $\overline{B}_s$ to f_i respectively. Denoting $\mathbf{A}_i = [A_i, \overline{A}_i]$ we will normalize the units of amplitude so that the decay rate to f_i for a given initial state $\Psi(0)$ is:

$$\Gamma_i(t) = |\mathbf{A}_i U(t)\Psi(0)|^2 \tag{12}$$

We can rewrite this as

$$\Gamma_i(t) = Tr\left[U^\dagger(t) \ R_i \ U(t) \ \rho_0\right] \tag{13}$$

where $\rho_0 = \Psi(0) \ \Psi(0)^\dagger$ and $R_i = \mathbf{A}_i^\dagger \mathbf{A}_i$ is the decay density matrix for a single quantum state.

Consider now, more generally a state F consisting of several individual quantum states: $F = \{f_i\}$. The decay rate as a function of time thus becomes:

$$\Gamma_F(t) = Tr\left[U^\dagger(t) \ R_F \ U(t) \ \rho_0\right] \tag{14}$$

where $R_F = \sum_i R_i$. In general R_F is Hermitian and thus can be written in the form:

$$R_F = \begin{bmatrix} u_F + v_F & w_F e^{i\theta_F} \\ w_F e^{-i\theta_F} & u_F - v_F \end{bmatrix} \tag{15}$$

where u_F, v_F and w_F are positive real numbers and $u_F^2 \geq v_F^2 + w_F^2$. If F consists of only a single quantum state then $u_F^2 = v_F^2 + w_F^2$.

Let us now expand Eqn. (14) for the initial states B_s, $\overline{B}_s$ and $\{B_s/\overline{B}_s\}$ which is an incoherent mixture of an equal number of B_s and $\overline{B}_s$. The corresponding density matrices for these initial states are:

$$\rho(B_s) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \rho(\overline{B}_s) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \rho(\{B_s, \overline{B}_s\}) = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{16}$$

The time dependent decay rates of these initial states to F in the limit $\eta \to 0$ (i.e. $|q/p| = 1$) are:

$$\begin{aligned}
\Gamma(B_s \to F) &= e^{-\Gamma_s t}\left[u_F C_{hy} + v_F C_x - w_F\left(S_{hy}\cos(\phi + \theta_F) - S_x\sin(\phi + \theta_F)\right)\right] \\
\Gamma(\overline{B}_s \to F) &= e^{-\Gamma_s t}\left[u_F C_{hy} - v_F C_x - w_F\left(S_{hy}\cos(\phi + \theta_F) + S_x\sin(\phi + \theta_F)\right)\right] \\
\Gamma(\{B_s/\overline{B}_s\} \to F) &= e^{-\Gamma_s t}\left[u_F C_{hy} - w_F S_{hy}\cos(\phi + \theta_F)\right]
\end{aligned} \tag{17}$$

where

$$x_s = \Delta m_s/\Gamma_s \qquad y_s = \Delta\Gamma_s/(2\Gamma_s) \tag{18}$$

and

$$C_x = \cos(x_s\Gamma_s t) \quad S_x = \sin(x_s\Gamma_s t) \quad C_{hy} = \cosh(y_s\Gamma_s t) \quad S_{hy} = \sinh(y_s\Gamma_s t) \tag{19}$$

The branching ratio to a particular final state is the time integral of the above. In particular we denote the branching ratio from the initial state $\{B_s/\overline{B}_s\}$ by

$$\hat{B}_F = \frac{u_F - w_F y_s \cos(\phi + \theta)}{(1 - y_s^2)\Gamma_s} \tag{20}$$

which is the average of the branching ratios of B_1 and B_2 to this final state.

Let us now extend the above formalism to the system of a correlated $B_s\overline{B}_s$ (i.e. produced at the $\Upsilon(5s)$) state where the meson 1 with momentum $\vec{k}$ decays to state F_1 and meson 2 with momentum $-\vec{k}$ decays to final state F_2. We can write time dependent decay rate as:

$$\Gamma^{\pm}_{F_1 F_2}(t_1, t_2) = S_{F_1 F_2} Tr\left[(U^\dagger(t_1)R_{F_1}U(t_1))Z_\pm(U^\dagger(t_2)R_{F_2}U(t_2))^T Z_\pm^\dagger\right] \tag{21}$$

where the superscript T stands for transpose, $S_{F_1 F_2}$ is a combinatorial factor; $S_{F_1 F_2} = 1$ if $F_1 \neq F_2$ and $S_{F_1 F_2} = \frac{1}{2}$ if $F_1 = F_2$ and

$$Z_\pm = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ \pm 1 & 0 \end{bmatrix} \tag{22}$$

is the matrix representation of the initial wave function, $\Psi^{C=\pm 1}$. Here t_1 is the time of decay for meson #1 and t_2 is the time of decay for meson #2.

Expanding the above for a $C = -1$ initial state, the result is:

$$\begin{aligned}
\Gamma^-_{F_1 F_2}(t_1, t_2) &= 2S_{F_1 F_2}e^{-\Gamma_s(t_1+t_2)}\big[(u_1 u_2 - w_1 w_2 C_1^\phi C_2^\phi)C_{hy}^- + (u_1 w_2 C_2^\phi - u_2 w_1 C_1^\phi)S_{hy}^- \\
&\quad + (v_2 w_1 S_1^\phi - v_1 w_2 S_2^\phi)S_x^- - (v_1 v_2 + w_1 w_2 S_1^\phi S_2^\phi)C_x^-\big]
\end{aligned} \tag{23}$$

while for an initial $C = +1$ state, the decay rate is:

$$\begin{aligned}
\Gamma^+_{F_1 F_2}(t_1, t_2) &= 2S_{F_1 F_2}e^{-\Gamma_s(t_1+t_2)}\big[(u_1 u_2 + w_1 w_2 C_1^\phi C_2^\phi)C_{hy}^+ - (u_2 w_1 C_1^\phi + u_1 w_2 C_2^\phi)S_{hy}^+ \\
&\quad + (v_2 w_1 S_1^\phi + v_1 w_2 S_2^\phi)S_x^+ - (v_1 v_2 - w_1 w_2 S_1^\phi S_2^\phi)C_x^+\big]
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
C_i^\phi &= \cos(\phi + \theta_i) & S_i^\phi &= \sin(\phi + \theta_i) \\
C_x^\pm &= \cos((t_1 \pm t_2)x_s\Gamma_s) & S_x^\pm &= \sin((t_1 \pm t_2)x_s\Gamma_s) \\
C_{hy}^\pm &= \cosh((t_1 \pm t_2)y_s\Gamma_s) & S_{hy}^\pm &= \sinh((t_1 \pm t_2)y_s\Gamma_s)
\end{aligned} \tag{25}$$

Integrating these results over t_1 and t_2 we obtain the correlated branching ratios

$$B^-(F_1 F_2) = \frac{2S_{F_1 F_2}}{\Gamma_s^2}\left[(u_1 u_2 - w_1 w_2 C_1^\phi C_2^\phi)\frac{1}{1 - y_s^2} - (v_1 v_2 + w_1 w_2 S_1^\phi S_2^\phi)\frac{1}{1 + x_s^2}\right] \tag{26}$$

$$\begin{aligned}
B^+(F_1 F_2) &= \frac{2S_{F_1 F_2}}{\Gamma_s^2}\Big[(u_1 u_2 + w_1 w_2 C_1^\phi C_2^\phi)\frac{1 + y_s^2}{(1 - y_s^2)^2} - (u_2 w_1 C_1^\phi + u_1 w_2 C_2^\phi)\frac{2y_s}{(1 - y_s^2)^2} \\
&\quad + (v_2 w_1 S_1^\phi + v_1 w_2 S_2^\phi)\frac{2x_s}{(1 + x_s^2)^2} + (v_1 v_2 - w_1 w_2 S_1^\phi S_2^\phi)\frac{1 - x_s^2}{(1 + x_s^2)^2}\Big]
\end{aligned} \tag{27}$$

where these are the ratios with respect to the total number of $B_s\overline{B}_s$ pairs produced in each of the two CP states. The fraction of $b\bar{b}$ events at the $\Upsilon(5s)$ peak will therefore be:

$$B_{5s}(F_1 F_2) = a\left((1 - r^*)B^-(F_1 F_2) + r^* B^+(F_1 F_2)\right) \tag{28}$$

where we assume that no attempt is made to distinguish between the $C = +1$ and $C = -1$ $B_s\overline{B}_s$ pairs. If these cases can be distinguished, for instance by counting the number of photons associated with the system, then an improvement in the statistics may be obtained although such a potential improvement is somewhat limited by the fact that at least 90% of the meson pairs are in the $C = -1$ state.

IV. INCLUSIVE AND EXCLUSIVE FINAL STATES

The key to obtaining basic physics parameters from the correlations and asymmetries we will discuss below is to choose final states where there is some a priori knowledge of the mixing strength w_F defined in Eqn. (15). To this end we will consider two opposite limits in which that is the case. On the one hand we will consider inclusive states which means a large fraction of the B_s decay modes. Depending on how you select such modes, w_F can be either 0 or related directly to y. On the other extreme CP eigenstates or related exclusive states which provide a case where $|w_F/u_F| = 1$.

Here we will consider two categories of inclusive final states, first of all flavor specific "taggable" final states which exclude quark content $c\bar{c}s\bar{s}$. Second of all "hadronic" final states which include $c\bar{c}s\bar{s}$.

The set of taggable decays includes all decays where the flavor of the meson can be determined from the decay products. For example in the decay $B_s \to \mu^+ \nu D_s^-$ it is known that at the instant of decay that there was a B_s (i.e. specifically a $\bar{b}s$ state) and not a $\overline{B}_s$ (i.e. $b\bar{s}$). We will denote taggable decays which indicate an initial state of B_s by $t+$ and taggable decays which indicate an initial state of $\overline{B}_s$ by $t-$. If we are not concerned with the flavor of the initial state we will denote the state as $t = (t+) \cup (t-)$. For such decays $w_t = 0$ since regardless of whether they tag $B = +1$ or $B = -1$ they cannot mix between B_s and $\overline{B}_s$.

These modes are a significant fraction of B_s decays. They consist of all semileptonic decays as well as most hadronic decays which do not have quark content $c\bar{c}s\bar{s}$. For instance we can include many hadronic decays containing only one charmed meson. It is advantageous to be able to include as many decays in the taggable sample as possible; we will assume that the tagging rate is 30% of all B_s decays.

If there is a partial rate asymmetry in taggable decays then $v_t \neq 0$. Initially we will not generally be considering observables which are particularly sensitive to this kind of effect.

The category of "hadronic" decays, which we denote "h" may include all hadronic decays or, more generally a subset of hadronic decays that includes decays with quark content $c\bar{c}s\bar{s}$. To optimize the utility of this sample, it is best to include all $c\bar{c}s\bar{s}$ final states and as few other hadronic states as possible into the "hadronic" sample. As we will show below, cuts which are tight enough to reduce the hadronic sample by about 20% while passing all $c\bar{c}s\bar{s}$ states greatly improve statistics in some cases.

For hadronic states $v_h \neq 0$ would indicate that there is a partial rate asymmetry. As with the taggable decays, the observables we discuss in this paper will generally not be sensitive to v_h so we will take $v_h = 0$. w_h however will be non-zero and is, in fact, tied to the B_s lifetime difference since the lifetime difference arises from a difference in the decay rate of the eigenstates to $c\bar{c}s\bar{s}$ states. Note that this also leads to a phase θ_h.

Exclusive flavor neutral states which consist of a single quantum amplitude, such as $F = D_s^+ D_s^-$, allow the direct measurement of the quantity $y\cos(\phi + \theta_i)$. For such states $w_F = \sqrt{u_F^2 - v_F^2}$ so assuming that the partial rate asymmetry is not large, $w_F \approx u_F$ is a good approximation. Thus, the matrix R_F depends only on u_F and the CP phase θ_F. In the limit of CP conservation this is further constrained. If F is a CP eigenstate and $CP = +1$, $u_F = w_F$ and $\theta_F = v_F = 0$ while if F is a $CP = -1$ state, $u_F = w_F$, $\theta_F = \pi$ and $v_F = 0$.

Other exclusive states such as $F = D_s^{+*} D_s^{-*}$ consist of multiple quantum states (in this case due to polarization). The parameter w_i is therefore not constrained. We may however be able to obtain information about the relative contribution of different amplitudes that make up F through the study of $\{B_s/\overline{B}_s\} \to F$. For instance in the case of $F = D_s^{+*} D_s^{-*}$ we can learn the contributions of the different polarization states through studies at a hadronic B_s experiment.

More generally, a semi-exclusive state consists of a small set of inclusive states such as $F = \psi + X$. Most likely there is no simple way to determine w_F or θ_F. However, if you combine information from time ordering asymmetries and time ordering-charge asymmetries discussed below, you can obtain the phase of such a decay. The Standard Model implies that all such phases will be small so if a large phase is discovered in any semi-inclusive set, this could be evidence for new physics.

V. TIME INDEPENDENT CORRELATIONS

Let us first consider the effect that mixing has on the time independent correlations between final states. If no mixing were present, the null hypothesis, the overall branching ratio to the state $F_i F_j$ would be given in a simple way by the product of the branching ratios for each of the B_s meson giving

$$B_{5s}^{null}(F_i F_j) = 2a S_{F_i F_j} \hat{B}(F_i) \hat{B}(F_j) \tag{29}$$

With mixing present, we will have a deviation of the measured value of B_{5s} from this expectation. The magnitude of this deviation will thus tell us about the mixing process.

In order to carry out this program, however you need to have an accurate value for the basic null hypothesis so $\hat{B}(F_i)$ and $\hat{B}(F_j)$ must be well determined. The systematic error in $B_{5s}^{null}(F_iF_j)$ is therefore likely to be the main limitation in using this technique to probe mixing.

Let us first consider the application to inclusive states and so apply Eqns. (20, 27 and 26) to t and h states.

In the case of taggable states, since $w_t = 0$,

$$\hat{B}_t = \frac{u_t}{\Gamma_s(1 - y_s^2)} \tag{30}$$

The origin of $\Delta\Gamma_s$ is the rate difference within the hadronic decays so that y is related to the hadronic decay mixing parameters by:

$$y_s\Gamma_s = w_h \cos(\phi + \theta_h) \tag{31}$$

it follows then that

$$\hat{B}_h = \frac{u_h - w_h y_s \cos(\phi + \theta_h)}{\Gamma_s(1 - y_s^2)} \tag{32}$$

It is convenient to write the correlated branching ratios strictly in terms of $\hat{B}$ since these are separately determined experimental quantities. So turning now to the correlated branching ratios for tt, th and hh final states we obtain.

$$
\begin{aligned}
B_{5s}(tt) &= a\hat{B}_t^2(1 - (1 - 2r^*)y_s^2) + O(x_s^{-2}) \\
B_{5s}(hh) &= a\hat{B}_h^2(1 - (1 - 2r^*)y_s^2(\hat{B}_h^{-1} - 1)^2)) + O(x_s^{-2}) \\
B_{5s}(th) &= 2a\hat{B}_t\hat{B}_h(1 + (1 - 2r^*)y_s^2(\hat{B}_h^{-1} - 1)) + O(x_s^{-2})
\end{aligned}
\tag{33}
$$

Here we drop the $O(x_s^{-2})$ terms since x_s is large for the B_s system.

In each of these cases, the correlated branching ratio depends only on y_s^2 and other measurable branching ratios and is independent of the CP violating phases. In Table I we show the number of $b\bar{b}$ events required at the $\Upsilon(5s)$ peak to give a $5-\sigma$ statistical deviation from $y = 0$ both in the case of $y_s = 0.1$ and $y_s = 0.05$. For the hadronic states, we also consider the scenarios where $\hat{B}_h = 0.7$ and $\hat{B}_h = 0.5$. From these results we see that applying cuts which reduce $\hat{B}_h$ from 0.7 to 0.5 will be very helpful in determining y_s with this strategy. For instance in the hh case, $N_{5s}(5\sigma)$ is lowered from 120×10^6 to 8×10^6. Note also that since hh, ht and tt correlations all measure y_s^2 it makes sense to combine the results from each of these combinations of final states. Combining data in this way will also lead to a reduction in $N_{5s}(5\sigma)$.

Thus there may well be adequate statistics to carry out this program, either presently or in the near future especially since BELLE as mentioned previously, already has accumulated appreciable amount of data at the $\Upsilon(5s)$. However the fractional deviation of the B_{5s} from B_{5s}^{null} is $O(y_s^2) \approx 1\%$. This implies that the accuracy of the input values for $\hat{B}_t$ and $\hat{B}_h$ needs to be less than 1% for the signal to be observable.

In the limit of CP conservation where the states are CP eigenstates, Eqns. (33) can be understood in terms of a simple argument. Consider the case of $B_{5s}(tt)$. The decay *rate* of each of the eigenstates to taggable final states is the same so the *branching ratio* will be inversely proportional to the total decay rate. Thus

$$B(B_1 \to t) = \frac{u_t}{\Gamma_s(1 - y_s)} \qquad B(B_2 \to t) = \frac{u_t}{\Gamma_s(1 + y_s)} \tag{34}$$

so taking the average, we obtain Eqn. (30)

If the initial state is $C = -1$, then the two meson state is $(|B_1\rangle|B_2\rangle - |B_1\rangle|B_2\rangle)/\sqrt{2}$ so that $B^-(tt) = B(B_1 \to t)B(B_2 \to t)$. Likewise, the $C = +1$ state is $(|B_1\rangle|B_1\rangle + |B_2\rangle|B_2\rangle)/\sqrt{2}$ so $B^+(tt) = \frac{1}{2}(B(B_1 \to t)B(B_1 \to t) + B(B_2 \to t)B(B_2 \to t))$. Putting in the eigenstate branching ratios Eqn. (34) into these expressions we can thus derive the tt correlation in Eqn. (33).

To obtain the other expressions in this limit, the hadronic branching ratio of the two CP eigenstates is

$$B(B_1 \to h) = \frac{u_h - y_s\Gamma_s}{\Gamma_s(1 - y_s)} \qquad B(B_2 \to h) = \frac{u_h + y_s\Gamma_s}{\Gamma_s(1 + y_s)} \tag{35}$$

which leads to the other two correlations in Eqn. (33).

Let us now consider the correlations involving an exclusive or semi-exclusive final state Y. For such a final state, let us define the quantity

$$P_Y = \frac{w_Y \cos(\phi + \theta_Y)}{u_Y} \tag{36}$$

which is the quantity that we wish to measure. It greatly simplifies the expressions below to define the related quantity:

$$\hat{P}_Y = \frac{P_Y - y_s}{1 - P_Y y_s} \tag{37}$$

In the limit that CP is conserved, $\hat{P}_Y$ is

$$\hat{P}_Y = \frac{\hat{B}(Y^+) - \hat{B}(Y^-)}{\hat{B}(Y)} \tag{38}$$

where Y^+ is the subset of Y which is CP $= +1$ and Y^- is the subset of Y which is CP $= -1$. More generally, for taggable states, $P_t = 0$ while for hadronic states $P_h = y_s \hat{B}_h^{-1}/(1 - y_s^2)$; thus:

$$\hat{P}_t = -y_s \quad \hat{P}_h = (\hat{B}_h^{-1} - 1)y_s \tag{39}$$

Using this notation for the branching ratio to the state Y:

$$\hat{B}_Y = \frac{1 - y_s P_Y}{1 - y_s^2} \frac{u_Y}{\Gamma_s} \tag{40}$$

The correlation between Y and the h and t inclusive states are:

$$B_{5s}(tY) = 2a\hat{B}_t \hat{B}_Y \left(1 + (1 - 2r^*)y_s \hat{P}_Y\right) + O(x_s^{-2}) \tag{41}$$

$$B_{5s}(hY) = 2a\hat{B}_h \hat{B}_Y \left(1 - (1 - 2r^*)y_s(\hat{B}_h^{-1} - 1)\hat{P}_Y\right) + O(x_s^{-2}) \tag{42}$$

The correlation between two different states in general is:

$$B_{5s}(Y_i Y_j) = 2a S_{Y_i Y_J} \hat{B}_{Y_i} \hat{B}_{Y_j} \left(1 - (1 - 2r^*)\hat{P}_{Y_i} \hat{P}_{Y_j}\right) + O(x_s^{-2}) \tag{43}$$

As discussed in[22, 23], it is easy to understand these correlations in the limit of CP conservation. In particular, suppose that Y is a CP$=+1$ eigenstate so $\hat{P}_Y = +1$. If we start with an initial charge conjugation -1 $B_s \overline{B}_s$ state, then if one of the mesons decays to Y, the other must therefore be in the B_2 state. The probability of it decaying to a taggable state is therefore $B(B_2 \to t)$ given in Eqn. (34). Conversely if we start with an initial charge conjugation $+1$ $B_s \overline{B}_s$ state, then if one of the mesons decays to Y, the other must therefore be in the B_1 state. In this case the probability of decaying to a taggable decay is $B(B_1 \to t)$. From this we can derive the tY correlation above in the case $\hat{P}_Y = 1$. We can generalize this argument to a case where $|\hat{P}_Y| < 1$ considering separately the Y^+ and Y^- components.

In summary then the correlations between inclusive states in Eqn. (33) can determine $|y_s|$ while the correlations between Y and h or t give $\hat{P}_Y$ which in turn gives $y w_Y \cos(\phi + \theta_Y)$. If w_Y can be determined in some way, then the cosine of the phase is determined.

If Y is flavor neutral exclusive state (i.e. a CP eigenstate), then $w_Y = u_Y$. For states with multiple amplitudes such as $D_s^* D_s^*$, $\psi\phi$, $D_s D_s \eta'$, w must be determined from detailed analysis of the final state or the state needs to be separated into its constituent quantum states, for instance by polarization or Dalitz plot analysis. For semi-exclusive states such as $\phi + X$ there is no way with this kind of data to factor P_Y into w_Y and $\cos(\theta_Y + \phi)$, so more information is required to do this.

Final State	Inputs	$N_{5s}(5\sigma)/10^6$ $(y=0.1)$	$N_{5s}(5\sigma)/10^6$ $(y=0.05)$
$\Upsilon(5s) \to tt$	$\hat{B}_t = 0.3,\ r^* = 0.1$	21	350
$\Upsilon(5s) \to ht$	$\hat{B}_h = 0.7,\ \hat{B}_t = 0.3,\ r^* = 0.1$	25	410
$\Upsilon(5s) \to ht$	$\hat{B}_h = 0.5,\ \hat{B}_t = 0.3,\ r^* = 0.1$	6.5	100
$\Upsilon(5s) \to hh$	$\hat{B}_h = 0.7,\ r^* = 0.1$	120	1900
$\Upsilon(5s) \to hh$	$\hat{B}_h = 0.5,\ r^* = 0.1$	7.8	130
$\Upsilon(5s) \to hh;\ ht;\ tt$	$\hat{B}_h = 0.7,\ r^* = 0.1$	16	260
$\Upsilon(5s) \to hh;\ ht;\ tt$	$\hat{B}_h = 0.5,\ r^* = 0.1$	4.9	78
$\Upsilon(5s) \to Yt$	$\hat{B}_t = 0.3,\ \hat{B}_Y = 10^{-3},\ P_Y = 1,\ r^* = 0.1$	33	130
$\Upsilon(5s) \to Yh$	$\hat{B}_h = 0.7,\ \hat{B}_Y = 10^{-3},\ P_Y = 1,\ r^* = 0.1$	76	300
$\Upsilon(5s) \to Yh$	$\hat{B}_h = 0.5,\ \hat{B}_Y = 10^{-3},\ P_Y = 1,\ r^* = 0.1$	20	78

TABLE I: The value of $N_{5s}(5\sigma)$, the number of $b\bar{b}$ events at the $\Upsilon(5s)$ required to observe a 5σ deviation from the null hypothesis Eqn. 29, *assuming perfect knowledge of the input branching ratios,* for the pairs of final states indicated. The results are shown for $y_s = 0.1$ and $y_s = 0.05$.

Final State	$\hat{B}_i/\hat{B}_j$	$\alpha_i/\alpha_j(\%)$	$N_{5s}(5\sigma)$ (10^9)
$D_s^+ D_s^-/D_s^+ D_s^-$	$1.1 \times 10^{-2}/1.1 \times 10^{-2}$	1/1	12
$K^+ K^-/K^+ K^-$	$3.3 \times 10^{-5}/3.3 \times 10^{-5}$	100/100	130
$D_s^+ D_s^-/K^+ K^-$	$1.1 \times 10^{-2}/3.3 \times 10^{-5}$	1/100	20
$\psi\eta/\psi\eta$	$9 \times 10^{-4}/9 \times 10^{-4}$	4/4	110
$\pi^0 K_s K_s/\pi^0 K_s K_s$	$3.7 \times 10^{-5}/3.7 \times 10^{-5}$	25/25	1600
$\psi\eta/\pi^0 K_s K_s$	$9 \times 10^{-4}/3.7 \times 10^{-5}$	4/25	210

TABLE II: The number of $b\bar{b}$ events (unlike the previous Table, here in units of 10^9) required to distinguish between the maximum and minimum possible correlation between two CP eigenstates at the 5σ level including the acceptances for the final states shown.

Correlations between two semi-exclusive states may also be used to determine P_Y by using the correlation in Eqn. (43).

In the case where the two states are the same or have the same CP eigenvalue this method has the advantage that it is almost a null experiment and it is therefore not subject to the contamination due to systematic errors in the input branching ratios. Looking at Eqn. (43) we see that in the limit of $r^* \to 0$ and $\hat{P}_{Y_i} = \hat{P}_{Y_j} = \pm 1$ which would be the case if CP were conserved then B_{5s} would be 0. This limit can be understood in terms of Bose statistics. In the $C = -1$ state, a B_s pair consists of one CP=+1 (i.e. B_1) and one CP=-1 (i.e. B_2) meson so you would never see two decays to the same CP eigenstate.

This correlation is not directly sensitive to y but it is sensitive to the mixing angle of the two states. As an illustration, let us consider the case where we have two states with $\hat{B} = 10^{-3}$ where $r^* = 0.1$ that are CP=+1 eigenstates. If there is no CP violation so $\hat{P}_{Y_i} = \hat{P}_{Y_j} = +1$ then B_{5s}(no mixing) $= 8 \times 10^{-8}$; where the fact that it is non-zero is due to the term proportional to r^*. On the other hand, suppose that there were large mixings in one or both of the channels so that $|\hat{P}_{Y_i}\hat{P}_{Y_j}| << 1$ and so B_{5s}(large mixing) $= 4 \times 10^{-7}$ about 5 times larger. Thus if you have $N = 7.5 \times 10^7$ events you can rule out the large mixing scenario at 5σ.

Most likely, however, one can probably not find a CP eigenstate decay mode with a branching ratio this high if the acceptance is factored in. In Table II we consider the number of $b\bar{b}$ required to distinguish between the minimum and maximum possible correlations at the 5σ level. The CP=+1 states we consider in particular are $D_s^+ D_s^-$ where we assume that the acceptance of this final state is $\alpha = 1\%$, and $K^+ K^-$ which should have acceptance nearly $\alpha = 100\%$. The latter state has the advantage that it has a significant penguin contribution and so is more likely to be influenced by new physics. The CP=-1 states we consider in particular are $\psi\eta$ where we assume that the acceptance of this final state is $\alpha = 4\%$, and $\pi^0 K_s K_s$ where we assume the acceptance is $\alpha = 25\%$.

We can see that the numbers are large even for super B-factories. Perhaps the most promising cases are $D_s^+ D_s^-/D_s^+ D_s^-$ and $D_s^+ D_s^-/K^+ K^-$ where the limiting factor is the acceptance of the $D_s^+ D_s^-$.

VI.   TIME DEPENDENT EFFECTS

Looking at Eqns. (23, 24) a couple of features of the time dependence are apparent. Let us denote $t_\pm = t_1 \pm t_2$ so that the expression for $\Gamma^-(t_1, t_2)$ is of the form $e^{-(\Gamma_s t_+)} f(t_-)$. If we integrate this over t_+ we obtain

$$\frac{dB^-}{dt_-} = \frac{1}{2\Gamma_s}e^{-\Gamma_s|t_-|}f(t_-). \tag{44}$$

where t_- ranges from $-\infty$ to $+\infty$ and the superscript $(-)$ on B indicates a C-odd initial state.

The fact that t_+ integrates out trivially is, of course, exploited in the design of the B-factory. We can determine the function f by binning events according to t_- and taking into account the exponential prefactor. Because the B mesons are created with proper motion in the lab frame, t_- can be inferred by the physical separation between the two decay vertices. In particular, the original interaction vertex where the mesons are created need not be determined. This is a useful feature of Γ^- since the e^+e^- interaction vertex cannot be directly observed.

In contrast, the expression for $\Gamma^+(t_1, t_2)$ is of the form $e^{-\Gamma_s t_+}g(t_+)$. If we integrate out the variable t_- we obtain

$$\frac{dB^+}{dt_+} = t_+ e^{-\Gamma_s t_+}g(t_+). \tag{45}$$

where t_+ ranges from 0 to $+\infty$. To determine t_+ we do indeed need to know the location of the interaction vertex. It is thus more difficult to study the time dependence of Γ^+. In fact experimental studies of the time dependence will be greatly helped by the feature that the meson pair is in a $C = -1$ more than 90% of the time because the time difference (t_-) is easier to measure than the time sum (t_+) at an asymmetric B factory.

The terms proportional to S_{hy}^- and S_x^- in Eqn. (23) have the property that they are antisymmetric under $t_1 \leftrightarrow t_2$. An observable which has the same symmetry will therefore be sensitive to these terms. The $S_x^- = \sin(\Delta m_s(t_1 - t_2))$ term oscillates at the rate Δm_s so it is not readily observable at B factories by directly resolving the oscillations. However, the asymmetry A'_{tj} discussed below does offer the prospect of sensitivity to this kind of time dependence.

The simplest such observable is the time ordering asymmetry:

$$A_{ij} = \begin{cases} +1 \text{ if } t_1 > t_2 \\ -1 \text{ if } t_1 < t_2 \end{cases} \tag{46}$$

the expectation value of A_{ij} will receive contributions from the coefficients of S_{hy}^- and S_x^- which we denote:

$$Q_{ij}^y = 2(u_i w_j C_j^\phi - u_j w_i C_i^\phi) = 2\Gamma_s^2 \hat{B}_i \hat{B}_j (1 - y^2)(\hat{P}_j - \hat{P}_i) \qquad Q_{ij}^x = 2(v_j w_i C_i^\phi - v_i w_j C_j^\phi) \tag{47}$$

where for flavor neutral final states with $v_i = v_j = 0$, the term proportional to Q_{ij}^x vanishes.

The expectation value of A_{ij} is, of course, just the asymmetry between the two decay modes according to which decays first. Since the two meson final state at B factories has a proper motion in the lab, this asymmetry can be calculated by:

$$< A_{ij} > = \frac{\text{(cases where F}_i \text{ happens later)} - \text{(cases where F}_j \text{ happens later)}}{\text{all F}_i\text{F}_j \text{ events}} \tag{48}$$

where "happens later" translates into "decays further downstream".

Evaluating this expectation value we obtain in the flavor neutral case:

$$< A_{ij} >= (1 - r^*)\left(Q_{ij}^y \frac{y_s}{1 - y_s^2}\right) = (1 - r^*)(\hat{P}_j - \hat{P}_i)y_s \tag{49}$$

The number of $b\bar{b}$ events at the $\Upsilon(5s)$ required to observe this asymmetry with a $n - \sigma$ significance is

$$N_{5s}(n\sigma) \approx \frac{n^2}{A^2} \frac{1}{2a\hat{B}_i\hat{B}_j} + O(y_s^2) = \frac{n^2}{2a(1 - r^*)^2 \hat{B}_i\hat{B}_j(\hat{P}_j - \hat{P}_i)^2 y_s^2} + O(y_s^2) \tag{50}$$

Let us now specialize this formula to the inclusive states. Since the two final states must be different to form this asymmetry, for a th final state this becomes:

$$N_{5s}(n\sigma; A_{ij}(th)) = \frac{n^2 \hat{B}_h}{2a(1 - r^*)^2 \hat{B}_t y_s^4} \tag{51}$$

where the dependence on $\hat{B}_h$ in the numerator comes from the expression for $\hat{P}_h$ in Eqn. 39.

Likewise, if we correlate semi-inclusive state Y with t or h states:

$$N_{5s}(n\sigma; tY) = \frac{n^2}{2a(1 - r^*)^2 \hat{B}_t \hat{B}_Y (\hat{P}_Y + y_s)^2 y_s^2} \tag{52}$$

$$N_{5s}(n\sigma; hY) = \frac{n^2}{2a(1 - r^*)^2 \hat{B}_h \hat{B}_Y (\hat{P}_Y + y_s(\hat{B}_h^{-1} - 1))^2 y_s^2} \tag{53}$$

In Table III we show the results for $N_{5s}(5\sigma)$. Again we consider a generic state Y with branching ratio 10^{-3} and $\hat{P}_Y$ near 1.

Looking at the value of $N_{5s}(5\sigma)$ for the asymmetry in the ht final state, we see that the asymmetry A_{ij} requires lower statistics to measure y_s. This is largely due to the fact that the taggable decay is more likely for the long lived B_2 state while the hadronic decay is more likely for the short lived B_1 state and the two tendencies combine in the asymmetry since the dominant $C = -1$ state is always $B_1 B_2$ by Bose statistics.

Furthermore, this is a null experiment, absent mixing the asymmetry will be 0 and so there is no large systematic error brought in due to the uncertainty of the input branching ratios.

Using the method of [24] one can devise an observable which measures this term with optimal statistical efficiency. If we neglect terms of $O(r^*)$ and $O(x_s^{-2})$ then the optimal observable to measure the S_{hy}^- term is

$$T_y^- = \tanh(t_1 - t_2) \tag{54}$$

This time dependence is proportional to the ratio between the time ordering asymmetric term in Eqn. (23) ($\propto S_{hy}$) and the time ordering symmetric term ($\propto C_{hy}$). For small y_s this observable offers some improvement, about factor of $O(2)$, over the unweighted time order asymmetry.

Consider now the case where taggable states are correlated with some other flavor neutral state, either h or Y. In this case we can take into account the sign of the tag and define the CP odd time ordering-charge asymmetry

$$A'_{tj} = B A_{tj} \tag{55}$$

where B is ± 1 for $t\pm$ taggable states, $i.e.$ events where the tagging decay indicates a B_s are weighted $+1$ while the events where the tagging decay indicates a $\overline{B}_s$ are weighted -1.

Since $v_{t+} = u_{t+} = u_{t-} = -v_{t-}$, this asymmetry is sensitive to Q_{tj}^x. Looking at the definitions of Q_{tj}^x and Q_{tj}^y we see that

$$Q_{(t\pm)j}^x = \pm \tan(\phi + \theta_j) Q_{(t\pm)j}^y \tag{56}$$

therefore

$$< A_{tj} > = (1 - r^*)(\hat{P}_j + y_s)y_s \tag{57}$$

$$< A'_{tj} > = (1 - r^*)(\hat{P}_j + y_s)\frac{(1 - y_s^2)x_s}{1 + x_s^2} \tan(\phi + \theta_j) \tag{58}$$

If both of these asymmetries are measured then $\tan(\phi + \theta_j)$ can be determined from the ratio without having to separately measure w_j. In practice the Standard Model prediction for $\tan(\theta + \phi) \approx .02$ so this method will generally bound (or discover) large phases due to the presence of new physics.

There are three distinct ways in which this pair of asymmetries (A and A') may be used.

1. With the inclusive states, A_{th} can be used to determine $|y_s|$. A'_{th} can be used to find $\tan(\phi + \theta_h)$.

2. With an exclusive state Y, A_{tY} determines $\cos(\phi + \theta_Y)$ (see Eqns. 36 and 37) and then A'_{tY} separately gives $\tan(\phi + \theta_Y)$.

3. For a semi-exclusive state such as $Y = \phi + X$ or $Y = \psi + X$ obtain $\tan(\phi + \theta_Y)$ from the ratio of A_{tY} and A'_{tY}.

In all cases of phase measurement, a large phase or discrepancies in phase between different modes indicates new physics.

In Table III we show some sample calculations of the asymmetries that might be seen in various decay modes and the statistics required to obtain a 5σ signal for the asymmetry. For each combination of modes we use an assumed acceptance α and give product branching ratio (factoring in α) $2a\alpha\hat{B}_i\hat{B}_j$ with respect to the total number of $b\bar{b}$ events at the $\Upsilon(5s)$ peak. We then give an estimated time ordering asymmetry A_{ij} which allows us to calculate the number of events required for a 5σ signal.

For combinations with taggable decays, we can use Eqn. (58) and estimate the time ordering-charge asymmetry A'_{ij} assuming $\tan(\phi + \theta_i) = 1$ and the corresponding $N_{5s}(5\sigma)$.

We can compare these $N_{5s}(5\sigma)$ values for the two asymmetries with the $\sim 1 \times 10^8$ which could be typical of current B-factories and 1.5×10^9 for a 5 ab^{-1} super B-factory (i.e. assuming a 50 ab^{-1} luminosity with about 10% of the running devoted to the $\Upsilon(5s)$).

For the case of the inclusive combination ht both asymmetries may be within the range of current B-factories. We have also included combinations of inclusive modes with exclusive modes which would likely require a super B factory.

First of all there is the exclusive mode $D_s D_s$ which, in the standard model, should be sensitive to the same phase as the ht combination. We have considered it with a 10% and a 1% acceptance where in the latter case somewhat more than 10^8 $b\bar{b}$ events are required. The $K^+ K^-$ mode considered would have both tree and penguin contributions hence new physics in a QCD penguin could contribute there. The case of $\psi\phi$ has already been studied through oscillations at D0 and CDF. Polarization analysis is helpful in separating the CP even from CP odd amplitudes. In the cases where we consider inclusive and exclusive modes with $(K_s\pi^0)_D$, i.e. a D_0 which specifically decays to $K_s\pi^0$, the final state connects D^0 and $\overline{D}^0$ so in the Standard Model there is sensitivity to the CKM phase γ. The more inclusive states $\psi + X$ and $\phi + X$ have larger branching ratios and the phase in those modes should agree with the overall mixing within the SM. A discrepancy of such phases would therefore indicate New Physics.

VII. CONCLUSION

In conclusion, with a sample of $O(10^7 - 10^8)$ $b\bar{b}$ events at the $\Upsilon(5s)$ peak there is the prospect of making a precision determination of $\Delta\Gamma_s/\Gamma_s$ through the study of tt, hh and th correlations. This will allow for the testing of the Standard Model prediction of the width difference.

Time independent correlations between various combinations taggable and hadronic decays have the disadvantage that there is a large systematic error originating from the input branching ratios. This can be remedied by using time dependent observables. One promising observable to use is the time ordering asymmetry between hadronic and taggable decays. The time ordering-charge asymmetry, which in addition to time ordering requires distinguishing B_s from $\overline{B}_s$, also can constrain the mixing phase although much more statistics would be needed to measure the expected Standard Model value.

At super B factories with about fifty times more luminosity, it becomes feasible to consider time ordering and time ordering-charge asymmetries with exclusive states and taggable or hadronic decays. Choosing specific exclusive decay modes can thus target different physics issues.

Time independent correlations between two exclusive CP eigenstates are not subject to the large systematic errors. For branching ratios about 10^{-3} these correlations can be sensitive to large CP-phases with B factory statistics.

[1] O. Aquines *et al.* [CLEO Collaboration], Phys. Rev. Lett. **96**, 152001 (2006) [arXiv:hep-ex/0601044].
[2] R. Louvot *et al.* [Belle Collaboration], Phys. Rev. Lett. **102**, 021801 (2009) [arXiv:0809.2526 [hep-ex]].

Final State (F_i/F_j)	α=Acceptance(%)	$2a\alpha\hat{B}_i\hat{B}_j$ (10^{-6})	A_{ij} (%)	$N_{5s}(5\sigma; A)$ (10^6)	A'_{ij} $(\tan(\phi+\theta)=1)$ (%)	$N_{5s}(5\sigma; A')$ (10^6)
h/t	100%	84000	1.3	1.8	0.49	12.6
$D_s^+ D_s^-/t$	10 (1)	120 (12)	10	21 (210)	3.78	150 (1500)
$D_s^+ D_s^-/h$	10 (1)	280 (28)	8.6	12 (120)		
$K^+ K^-/t$	100	4.0	10	630	3.8	4400
$K^+ K^-/h$	100	9.2	8.6	367		
$\psi\phi/t$	10	11	5.6	725	2.1	5100
$\psi\phi/h$	10	26	4.8	417		
$(K_s\pi^0)_D\phi/t$	10	0.37	10	6800	3.8	47000
$(K_s\pi^0)_D\phi/h$	10	0.84	10	3000		
$\psi + X/t$	10	140	1	1800	0.38	13000
$\psi + X/h$	10	325	1	770		
$\phi + X/t$	50	1700	1	150	0.38	1000
$\phi + X/h$	50	3900	1	63		
$(K_s\pi^0)_D + X/t$	10	923	1	271	0.38	1900
$(K_s\pi^0)_D + X/h$	10	2200	1	116		
$\phi\gamma/t$	49	2.7	10	37	3.8	260
$\phi\gamma/h$	49	6.3	10	16		

TABLE III: This table shows an estimate of the requirement for observing the time ordering and time ordering-charge asymmetry of the given pairs of final states shown in the first column. For each final state pair, we have assumed the acceptance α. Note that for the D_s pairs we have taken a 10% and a 1% scenario (shown in brackets). In the third column we show the effective branching ratio compared to $e^+e^- \to b\bar{b}$. In the A_{ij} column we show the estimated time ordering asymmetry assuming that $\cos(\phi+\theta) \approx 1$ as expected in the Standard Model. For the CP eigenstate, $D_s^+ D_s^-$ and $K^+ K^-$ we assume that $|P| = 1$, i.e. no large direct CP violation in the decay. For the inclusive states $\psi + X$ and $\phi + X$ we suppose the value of $\hat{P} \approx 10\%$. The next column shows $N_{5s}(5\sigma)$, the number of $b\bar{b}$ events needed to observe the given time ordering asymmetry with 5σ statistics. For the cases where a decay is with a tagged decay, we give the time ordering-charge asymmetry $\hat{A}_{ij}$ assuming that the total phase is $\tan(\phi+\theta) = 1$ as might be the case in New Physics. Likewise, we also give the number of $b\bar{b}$ events needed to observe this time ordering-charge asymmetry with 5σ significance.

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